



Strategy of asset portfolio risk diversification through value drivers

Francisco Roberto Farias Guimarães Júnior^[a], Charles Ulises De Montreuil Carmona^[b], Luciana Gondim de Almeida Guimarães^[c]

^[a] Doutor em Administração pela Universidade Federal de Pernambuco (PROPAD-UFPE). Professor na Universidade Federal da Paraíba, João Pessoa, PB – Brasil, email: rguimaraesjr@ccsa.ufpb.br

^[b] Doutor em Engenharia de Produção pela Pontifícia Universidade Católica do Rio de Janeiro (PUC-RIO). Professor na Universidade Federal de Pernambuco, Recife, PE – Brasil, email: carmona@ufpe.br

^[c] Doutora em Administração pela Universidade Federal de Pernambuco (PROPAD-UFPE). Professora na Universidade Potiguar, Natal, RN – Brasil, email: luciana.almeida@unp.br

Abstract

The risk diversification of an asset portfolio of investments is underlying in the idea that all securities have an idiosyncratic behavior which allows compensating a specific stock loss by the gain achieved by other stock into the portfolio. However, we know that the portfolio selection process should excel for choosing assets capable of creating and generating value on the long term. Thus, the objective of this research was to verify if the portfolios selected through their value drivers present the diversification benefits that were determined in prior researches. We had used the data available at Economica data base of the following Stock Exchanges: Argentina; Brazil; Chile; and Mexico. To select the portfolios by value drivers we used a model based upon the weighted factors decision matrix where the securities were hierarchized by their grades. The variables used as factors, were the Tobin's Q, Beta, Leverage, Price/Earning Ratio, and the Price Sales Ratio. All portfolios were compared with that selected through Markowitz (1952) model. The results show us that the portfolios selected through value drivers have obtained the benefits of the diversification process convergent with prior researches. On the other hand, we verified that the stocks amount into portfolios constructed through Markowitz (1952) model have had high positive correlation with the stocks amount in the Stock Exchange what resulted in portfolios with 44 assets, for instance. For future studies we suggest: the using of generalized linear model instead the multiple regressions to figure out the factor weights; to use others fundamentalist variables; to apply this study in other Stock Exchanges.

Keywords: Diversification; Value Drivers; Asset Portfolio Investment.

Introduction

For over 50 years, researchers have been studying the portfolio diversification for different purposes and different goals. The Evans and Archer (1968) research paper was one of the first to examine how many stocks are sufficient to diversify a portfolio, so that they could completely remove the portion of the idiosyncratic risk of each asset. This research “examined the rate at which the variation of returns for randomly selected portfolios is reduced as a function of the number of securities included in the portfolio” (EVANS; ARCHER, 1968, p 761.).

The results of Evans and Archer (1968, p. 766) reported that “most of the unsystematic variation is eliminated by the time the 8th security is added to the portfolio”. This observation was supported by *t* and *F* tests which indicated that only with a substantial increase in a portfolio compounded by eight securities is necessary for a significant reduction in the average of the standard deviations and in the mean values of the dispersions.

In their tests, Evans and Archer (1968) found out that for a portfolio with only two securities, adding another security can causes a significant reduction on the portfolio volatility at the level of 0.05. For portfolios with securities titles, it was necessary the inclusion of five bonds to obtain the same reduction in the portfolio volatility. In portfolios compounded by 16, it was necessary to include more 19 different securities and for portfolios with more than 19 papers “no significant reduction was possible within the range of the analysis, which was 40 securities” (EVANS; ARCHER, 1968, p. 766).

Several research studies have complemented Evans and Archer (1968). Fisher and Loire (1970) studied the effect of diversification in portfolios randomly elaborated and structured through a combination of stocks of different economy sectors. Wagner and Lau (1971) associated the value of the coefficient of determination R^2 to the systematic risk and studied the diversification effect. In their results, they found that in portfolios with more than 10 securities, reducing the idiosyncratic risk ($1 - R^2$) is insignificant.

In Brazil, Brito (1989) found that the benefits of diversification could be achieved with a portfolio of eight stocks and a portfolio of over 15 securities do not get a substantial reduction in its risk when compared with 15-securities one. Ceretta and Costa Jr. (2000) used the price information of 158 stocks which had belonged to Bovespa from January 1993 to December 1997 to verify that “with an equally weighted portfolio of 12 securities, the investor can get excellent results eliminating more than 52% of a typical stock risk and over 83% of the risk [idiosyncratic]” (CERETTA; COSTA JR, 2000, p. 32.).

A common point on these papers is the fact they have used a method of random securities choice by using simulation models to diversify the portfolio. But if securities are selected through its value drivers, such portfolios would have the benefits of diversification with the same amount of securities? Thus, the aim of this study was to determine how many securities are needed to diversify a portfolio when they are selected by their value drivers.

Diversification

The idea of diversification is underlying to the fact that the prices of financial securities are not perfectly correlated so that when several different securities are combined, a change in an individual price can be compensated by complementary changes in the others, reducing the total variation (risk) of the portfolio (MARKOWITZ, 1959).

The risk which can be eliminated through diversification is called own risk or unsystematic risk (idiosyncratic risk). In contrast, the systematic risk, also known as market risk, cannot be eliminated through diversification and influences the all prices' behavior and therefore affects all investors, regardless of the number of securities they held.

Markowitz (1959, p. 102) emphasized the importance of diversification in a portfolio and showed how, "in portfolios involving a large number of securities, the variance loses its importance when compared with the covariance". The mathematical model for the portfolio's variance of an equally weighted portfolio presented on Markowitz (1959, p. 111) clearly demonstrates that. Using a very large number of securities, the first term of the equation tends to zero, and the variance of the portfolio tends to the average covariance among the of all security market returns:

$$\text{portfolio variance} = \frac{\text{sum of the variances}}{N^2} + \frac{N-1}{N} \times \text{average covariance} \quad (1)$$

The development of this expression can also to be seen on Elton and Gruber (1977) which, unlike the empirical studies that preceded them, they had derived an analytical expression for the relationship between the size of a portfolio and its risk. Based on the variance of a portfolio of N securities:

$$\sigma_p^2 = \sum_{i=1}^N x_i^2 \sigma_i^2 + \sum_{i=1}^N \sum_{\substack{j=1 \\ j \neq i}}^N x_i x_j \text{cov}(A_i, A_j) \quad \forall i, \forall j \quad (2)$$

Where:

σ_p^2 = The portfolio variance;

x_i = Percentage (weight) of the security i in the portfolio p ;

σ_i^2 = Variance of the security i ;

x_j = Percentage (weight) of asset j in portfolio p ;

\forall = Mathematical symbol which means "for all", "for any", "for each";

$\text{cov}(A_i, A_j)$ = Covariance of the securities i and j returns.

One alternative to obtain a diversified, but not the smartest, is simply by choosing a N sufficiently large and dividing the total capital equally among existing securities, as showed in below:

$$x_i = \frac{1}{N}, \forall i$$

In this case, the variance of the portfolio can be expressed as:

$$\sigma_p^2 = \sum_{p=1}^N \left(\frac{1}{N}\right)^2 \sigma_i^2 + \sum_{i=1}^N \sum_{\substack{j=1 \\ j \neq i}}^N \left(\frac{1}{N}\right)^2 \text{cov}(A_i, A_j) \quad (3)$$

As for N assets there are $N(N-1)$ covariance pairs, we can express the expected average covariance (E) by:

$$E[\text{cov}(A_i, A_j)] = \frac{\sum_{i=1}^N \sum_{\substack{j=1 \\ j \neq i}}^N \text{cov}(A_i, A_j)}{N(N-1)} \Leftrightarrow \sum_{i=1}^N \sum_{\substack{j=1 \\ j \neq i}}^N \text{cov}(A_i, A_j) = N(N-1) \cdot E[\text{cov}(A_i, A_j)] \quad (4)$$

And the average expected variance (E) can be expressed as:

$$E[\sigma_i^2] = \frac{\sum_{i=1}^N \sigma_i^2}{N} \quad (5)$$

Substituting (4) and (5) in (3) we have:

$$\begin{aligned} \sigma_p^2 &= \frac{1}{N} \cdot E[\sigma_i^2] + \frac{1}{N^2} \cdot N(N-1) \cdot E[\text{cov}(A_i, A_j)] \\ \sigma_p^2 &= \frac{E[\sigma_i^2]}{N} + \frac{(N-1)}{N} \cdot E[\text{cov}(A_i, A_j)] \end{aligned}$$

Thus, it's possible to clearly observe that when N increases, the first part of the portfolio variance (which represents the own risk of the assets) tends to zero. However, the second remains:

$$\lim_{N \rightarrow \infty} \sigma_p^2 = \lim_{N \rightarrow \infty} \frac{E[\sigma_i^2]}{N} + \lim_{N \rightarrow \infty} \left\{ \frac{N-1}{N} \cdot E[\text{cov}(A_i, A_j)] \right\} = E[\text{cov}(A_i, A_j)] \quad (6)$$

This shows that even for a very large N there is always a residual variance in the portfolio variance which tends to the average assets covariance. Sharpe (1963) achieved a similar result using a single index model (CAPM), whose the expected asset return is given by:

$$R_i = \alpha_i + \beta_i \cdot (R_M - R_f) + \varepsilon_i \quad (7)$$

Where:

R_i = Expected return of an asset i ;

α_i = Vertical intercept of the security market line;

β_i = Sensitivity of the expected excess asset returns to the expected excess market return;

R_M = Expected return (average return) of the market;

R_f = Return of the risk-free rate;

ε_i = Random error.

Assuming R_f and α_i constant, the variance of the model is:

$$\sigma_i^2 = \beta_i^2 \sigma_M^2 + \sigma_{\varepsilon_i}^2 \quad (8)$$

In this case, the return of a N -assets portfolio is given by:

$$R_p = \sum_{i=1}^N x_i (\alpha_i + \beta_i \cdot (R_M - R_f) + \varepsilon_i) \quad (9)$$

$$R_p = \sum_{i=1}^N x_i \alpha_i + \sum_{i=1}^N x_i \beta_i \cdot (R_M - R_f) + \sum_{i=1}^N x_i \varepsilon_i \quad (10)$$

$$R_p = \alpha_p + \beta_p \cdot (R_M - R_f) + \varepsilon_p \quad (11)$$

Adopting R_p and α_p constant, we have:

$$\sigma_p^2 = \beta_p^2 \sigma_M^2 + \sigma_{\varepsilon_p}^2 \quad (12)$$

Whereas $\text{cov}(\varepsilon_i, \varepsilon_j) = 0, \forall i \neq j$, the variance of the ε_p reduces to:

$$\sigma_{\varepsilon_p}^2 = \sigma^2 \left(\sum_{i=1}^N x_i \varepsilon_i \right) = \sum_{i=1}^N x_i^2 \sigma_{\varepsilon_i}^2 + \sum_{i=1}^N \sum_{\substack{j=1 \\ j \neq i}}^N \text{cov}(\varepsilon_i, \varepsilon_j) = \sum_{i=1}^N x_i^2 \sigma_{\varepsilon_i}^2 \quad (13)$$

So the variance of the portfolio can be expressed as:

$$\sigma_p^2 = \left[\sum_{i=1}^N x_i \beta_i \right]^2 + \sum_{i=1}^N x_i^2 \sigma_{\varepsilon_i}^2 \quad (14)$$

Assuming that assets are equally distributed in the portfolio, we have:

$$\sigma_p^2 = \left[\sum_{i=1}^N \frac{1}{N} \beta_i \right]^2 + \sum_{i=1}^N \left(\frac{1}{N} \right)^2 \sigma_{\varepsilon_i}^2 \quad (15)$$

Noticing that:

$$E[\beta_i] = \frac{\sum_{i=1}^N \beta_i}{N}, \text{ and} \quad (16)$$

$$E[\sigma_{\varepsilon_i}^2] = \frac{\sum_{i=1}^N \sigma_{\varepsilon_i}^2}{N} \quad (17)$$

And substituting (16) and (17) into (15) we obtain:

$$\sigma_p^2 = E^2[\beta_i] + \frac{1}{N} E[\sigma_{\varepsilon_i}^2] \quad (18)$$

Finally, assuming that the limit of $N \rightarrow \infty$, it seems that the portion of the asset's own risks tends to zero. In this case, however, it is clear that the remaining portion, which cannot be eliminated through diversification, is closely linked to market index β_i (sensitivity of asset i to the market movements), which is the systematic risk (SHARPE 1963; EVANS; ARCHER, 1968). About that, Evans and Archer (1968, p. 761) have commented that

if the number of securities included in a portfolio were to approach the numbers of securities in the market, one would expect the variation of the portfolio return to approach the level of systematic variation – that is, the variation of the market return, suggesting a relationship which behaves as a decreasing asymptotic function.

Value Drivers

Ball and Brown (1968) have developed what they themselves defined being “perhaps the first attempt to assess empirically the relative importance of the annual income number in the stock prices” (BALL; BROWN, 1968, p 176.). At the beginning of the paper, the authors presuppose that “the limitation of a completely analytical approach to usefulness [of accounting practices] are illustrated by the argument that income numbers cannot be defined substantively, that they lack ‘meaning’ and are therefore of doubtful utility” (BALL; BROWN, 1968, p. 159).

In the sequence of their arguments, Ball and Brown (1968) picked out the researches of the Fama (1965), Samuelson (1965), Fama and Blume (1966), and Jensen (1968) to argue that recent developments in capital markets theory justifies the using of the behavior of asset prices as a test of the usefulness of accounting practices.

Influenced by Ball and Brown (1968), several studies sought to detect the relationship between annual income and stock returns. Hopwood and McKeown (1985) and Hoskin, Hughes and Ricks (1986) concluded that revenues do not have a major influence on stock returns. On other hand, Swaminathan and Weintrop (1991) Rees Sivaramakrishnan (2001) Ertmur, Livnat and Martikainen (2003) and Loch Court (1999), Liu, Nissim and Thomaz (2000) and Jegadesh and Livnat (2004) found that revenues have better explanatory power about the stock return than profits.

In Brazil, Paula Leite and Sanvicente (1990) studied the relationship between valuation multiples and stock returns. The null hypothesis of Paula Leite and Sanvicente (1990, p. 22) “indicate the absence of ‘information content’ as it corresponds to equal extraordinary return with and without information. The rejection of the null hypothesis is that would lead us to assume that the event could have a significant impact”. The study has found that among the market multiples, the ones who had explanatory power about the stock return were: price/earnings ratio, with a negative relationship, and sales/price ratio, with a positive relationship.

Costa Jr. and Neves (2000) studied the influence of the following fundamentalist variables (value drivers) in stock returns: price/earnings ratio, market value, book-to-market equity and the beta. The research was carried out with companies traded on Bovespa for the period 1987 to 1996 and the findings indicated: a negative relationship with the price/earnings ratio and market value and a positive relationship between the profitability and the book-to-market equity.

The negative coefficients of the price/earnings ratio and market value ob-

tained in this study confirm the results of Paula Leite and Sanvicente (1990) and Hazzan (1991) for the Brazilian capital market. In the Hazzan (1991) research, portfolios compounded of low price/earnings ratio tend to provide better performance than the high price/earnings ratio ones, even after risk adjustment.

However, despite the fact the value drivers analyzed in the Costa Jr. and Neves (2000) research have influence on the explanations of the changes in the assets average returns, the beta was strongly representative being the variable that stood out this explanation. Thus, based on the tests, the authors stated that the CAPM is unspecified due to the possibility of including other factors in the asset returns behavior, beyond the beta.

Nagano, Merlo and Silva (2003) conducted a study to determine whether other fundamental variables, in addition to the β , are important in explaining the asset returns variations. In the investigation, they evaluated all non-financial companies (55 securities) that made up part of the Ibovespa portfolio (market proxy) between May 1995 and May 2000. They have analyzed the following variables (market multiples) in relation to the stock returns: the β ; total of assets/book value; total of assets/market value; dividend/price; cash flow/price; market liquidity ratio; market value; book value/price; earning/price ratio; and sales/price ratio.

In general, it is easy to verify that researchers have found a significant relationship between economic and financial variables (value drivers) and the stock return, especially the ones which are components of the cash flow as: margin (gross and liquidity); revenue; tax rates; assets investments; and capital cost. However, in none of these studies the researchers sought to identify the benefits of the diversification in portfolios created based on value drivers.

Methodological Procedures

The first step of the research was to develop a model to create portfolios through value drivers. Then, we compared the results by the results of the Markowitz (1952) model. The model used is based on assets hierarchy and choose them one by one, in descending order, until the portfolio risk stabilizes. The portfolio risk was admitted stable when the relationship between the risk of the portfolio with n assets and portfolio risk with $n - 1$ assets was equal to $1 +$ or $- 0.05$, in a series of at least 10 observations. This method of analyzing the portfolio risk stabilization is simple and is different from the method used by Evans and Archer (1968), Statman (1987), Cereta and Costa Jr. (2000) and Sanvicente and Bellato (2004).

Evans and Archer (1968) carried out two tests to figure out the reduction of non-systematic risk, which have been replicated by Cereta and Costa, Jr. (2000): t tests in successive averages standard deviations that indicate of successive significance average increases in the portfolio sizes; and F tests in successive standard deviations compared to the mean of the standard deviations which indicate the convergence of individual observations in average values.

Sanvicente and Bellato (2004) used the same method of Statman (1987). In these studies the authors randomly selected portfolios from 1 toward n assets to create from 1 toward n investment portfolios, calculated their respective standard deviations and compared the benefits of the diversification with portfolio of 50 and

500 securities, respectively. To vary the portfolio risks of 50 and 500 securities, the authors used the risk-free asset in their compositions.

We do recognize the importance, relevance and robustness of the methods used by these surveys. However, because these methods are more complex than a simple relationship between the portfolio risk of an n -assets portfolio and the portfolio risk of $n - 1$ -assets one, which causes the same effect in the stabilization risk analysis, this model adopted this second option.

The chosen value drivers were: Tobin's Q, Beta; Leverage; Price/Earnings ratio; and Price/Sales ratio. Tobin's Q can be defined as the ratio between a physical asset's market value and its replacement value (Reinhart, 1977). The problem of theoretical Tobin's Q model is the determination of the company's debt value and physical asset's replacement value, given that we must use the fair value and not the accounting values (book values). The Lindenberg and Ross (1981) paper discloses a method to calculate the theoretical value Tobin's Q using available data in databases such as Economatica.

To assign the weight of each value driver, we decided to perform a cross-section multiple linear regression with β_0 equal to zero, between the stock returns in a given year (dependent variable) and the values of the value drivers of these stocks in the last trading session of that year (independent variables).

It is known that to force β_0 to zero, a matrix based on cross-product matrix is analyzed instead of the correlation matrix, which changes the set slope of the adjusted line and can affect the results. However, the goal of forcing the β_0 to zero is to get the same value of β_0 in all regressions performed and to prevent that the coefficient β_0 is the most significant coefficient in a given regression, for instance. This idea relies on some researches that use β_0 -equal-to-zero regression. The Cobb-Douglas, for example, relates the output (y) with the physical capital and labor. If there is a constant in this model (β_0), we can have an unreal capacity of manufacturing goods without resources when physical capital and labor have null value.

Chambers and Dunstan (1986) presented a research using the regression model between sugarcane crops and their harvests with constant equal to zero. Clearly, if no area is grown, no sugarcane is harvested. Casella (1983) applied the β_0 -equal-to-zero linear regression to study relative fuel consumption against the vehicle weight. Of course, if the vehicle weight is zero (no vehicle), there will be no fuel consumption. Adelman and Watkins (1994) applied linear regression with β_0 equal to zero to evaluate mineral deposits. The lack of mineral deposits involves no value.

Based on this same principle, the motivation to work with β_0 -equal-to-zero regression is logical. Thus, if there is no value drivers, implies that there is no stock. If there is no stock, it means that there is no price. Therefore, if there is any value to β_0 and the values of the value drivers are zero, the dependent variable (stock price) will be equal to β_0 , which is impossible, given that the absence of value drivers implies in absence of stock.

After the calculation of each $\hat{\beta}_i$, we used all $\hat{\beta}_i$ as the weight of each value driver (fundamentalist variable) to rank (hierarchy) the assets through the weighted sums of each asset ($SP At_i$) as the following manner:

$$SP At_i = \sum_{i=1}^n \hat{\beta}_i \times Value\ of\ VD_i\ At_i \quad (19)$$

Where:

- VD_i = Value Drivers;
- $\hat{\beta}_i$ = Weight of each value driver;
- $Value\ of\ VD_i\ At_i$ = Value of the value driver in each asset;
- $SP At_i$ = Weighted sum of each asset.

We warned that because of working with real data, it is possible to obtain negative values of weighted sums. For a hierarchy of assets, there is no problem in defining negative values. However, to determine the percentage of assets that will be part of the investment portfolios, the negative values will induce wrong values on determining the weight of each asset.

To figure out this problem, we must standardize the weighted sums, relating these with the total range of the weighted sum values. We can do that by calculating the ratio of the difference between the values of the weighted sum of the asset i and the weighted sum of the asset n against difference between weighted sum of the first asset and the weighted sum of the asset n , as the following equation:

$$SP' At_i = \frac{SP At_i - SP At_n}{SP At_1 - SP At_n} \quad (20)$$

Where:

- $SP' At_i$ = Standardized value of the weighted sum of asset i ;
- $SP At_i$ = Unstandardized value of the weighted sum of asset i ;
- $SP At_1$ = Unstandardized value of the weighted sum of the first-ranked asset;
- $SP At_n$ = Unstandardized value of the weighted sum of the last-ranked asset.

With the standardized values of the weighted sums, the next step is portfolio selection. The size of each portfolio is not fixed. The assets must be included, one by one, until the risk of the portfolio, calculated by dividing the standard deviation of portfolio returns, stabilizes. To determine the participation percentage of each asset in each of the investment portfolios we do as shown in the following procedure:

$$\%At_i = \frac{SP' At_i}{\sum_{i=1}^j SP' At_i} \quad (21)$$

Where:

- $\%At_i$ = Participation percentage of each asset in the portfolio;
- $SP' At_i$ = Standardized value of the weighted sum of asset i ;
- $\sum_{i=1}^j SP' At_i$ = Sum all standardized value of the weighted sum of asset which integrates the portfolio in such a way that j can assume values from 1 to n ;

The data used for the portfolio selection were the ones available on Economatica database to stock markets of the: Brazil; Argentina; Chile; and Mexico. The first filter used in the assets mining was the filter “type of asset”. In this option, we selected all stocks with the classification “stock and ADR, etc. (Foreign company)”. The second filter was the liquidity filter, which is given by the negotiability index and measures the relative stock participation in sale-and-buy operations on the Stock Exchange where it is traded. This is calculated by the following equation and this proxy was used by Xavier (2007), Bruni and Famá (1998) and Machado and Medeiros (2011).

$$NI = 100 \times \frac{p}{P} \times \sqrt{\frac{n}{N} \times \frac{v}{V}} \quad (22)$$

Where:

- p = Number of days in which there was at least one operation with the chosen stock within the chosen period;
- P = Total number of days with the chosen stock within the chosen period;
- n = Number of operation with the chosen stock within the chosen period;
- N = Number of operation with all stocks within the chosen period;
- v = Amount of money in with the chosen stock within the chosen period;
- V = Amount of money with all stocks within the chosen period.

After of applying the filters to asset selection, we extracted the values of each of the value drivers that were used for the portfolio selection. In the process of portfolio selecting for the year 1995, for example, we used the value drivers’ value on December 31, 1995 as independent variables, and the values of the annual stock returns as dependent variables on the cross-sectional regression. After application of the portfolio selection proposed model, we selected 20 assets of each country to construct the portfolios. The calculation each portfolio risk was done by calculating the standard deviation of daily returns of these portfolios, in 1995.

Then, portfolios with data from January of the following year (in this case 1996) to December 2012 were selected and their performances were compared. This same process was repeated in all other years, until 2011. For portfolios which were selected in 2011 we applied data from January 2012 to December 2012.

For Markowitz algorithm, the daily stock returns were used to determine what stocks and their weights would be chosen in each portfolio for each country. The adopted objective for the Markowitz model was to maximize the return on the portfolio and the imposed constraint was that the risk of the optimized portfolio should be equal to the risk of the portfolio selected by the proposed model. This same process was repeated every year until 2011. For portfolios selected in 2011, data from January 2012 to December 2012 were applied.

Data Analysis and Results Interpretation

Following what was proposed in the methodological procedures, we present the results of the performed analyzes, verifying the amount of assets in each portfolio that was selected through the model which uses the value drivers. Table 1 shows the

result of the assets quantity in these portfolios.

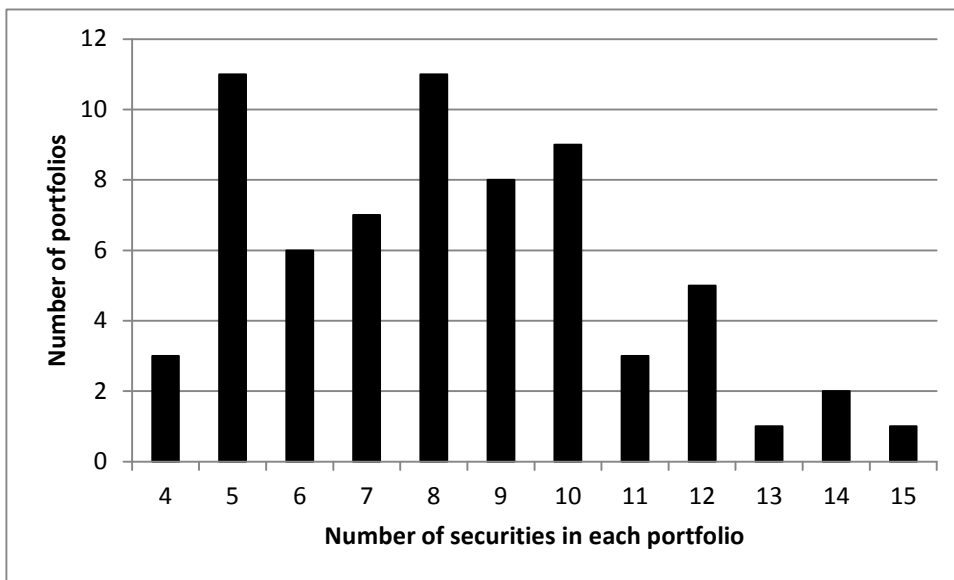
Table 1: Number of assets in the portfolios formed by the proposed model

Year of Portfolio Selection	BCBA	BCS	BMV	BOVESPA
1995	4	8		8
1996	5	12	9	8
1997	9	9	7	5
1998	8	5	9	14
1999	11	5	7	8
2000	10	10	8	6
2001	6	6	6	9
2002	4	11	8	8
2003	5	12	15	11
2004	14	8	9	10
2005	6	7	9	10
2006	12	9	12	10
2007	13	10	7	7
2008	5	10	12	4
2009	8	5	10	7
2010	5	6	5	8
2011	5	5	10	7

Source: Prepared by the author.

The Figure 1 shows a histogram of the Table 1. In the graph, the horizontal axis represents the information about the amount of assets in each portfolio and in the vertical one, the amount of portfolios.

Figure 1: Number of securities in each portfolio selected by the proposed model



The first finding is a lack of a standard. From a total of 67 portfolios, 22 were compounded by 5 and 8 securities (11 for each group); 9 were compounded by 10 securities; 8 portfolios were compounded by 9 securities; 7 portfolios by 7 securities; and 6 portfolios by 6 securities, which totalize 52 portfolios. The remaining portfolios

were compounded by 4, 11, 12, 13, 14 or 15 securities. These results converge to the results of previous studies which aimed to identify how many assets are necessary to eliminate idiosyncratic risk. Evans and Archer (1968) found that most of the idiosyncratic risk is eliminated after the addition of the 8th security in the portfolio. Lau and Wagner (1971) found that in portfolios with more than 10 stocks, the reduction of the idiosyncratic risk is negligible.

In Brazil, Brito (1989) found that the benefits of the diversification could be achieved with a portfolio with eight stocks and in portfolios with more than 15 securities the reduction of the diversifiable risk is noticeable. Ceretta and Costa Jr. (2000) found that portfolios with 12 stocks eliminate over 83% of the unsystematic risk. This is the first indicator that the model developed in this research can be used as an alternative to the portfolio selection.

For the portfolios selected by Markowitz (1952) model, the results are presented in Table 2. It is important to highlight that, the previous selection of the assets used in this model, which represents the “first stage” (Markowitz, 1952, p. 77), took into account only the daily liquidity. This means that all stocks that had at least one sale-buy operation per day were extracted to make part of the used list. It is known that practitioners interested in using this model invest more time and effort in a more rigorous first-stage selection process using some more robust criteria.

Note the absence of portfolio selected by the Markowitz model for the Argentina Stock Exchange (BCBA) in 2001. This occurred because only the stock of the Galicia Financial Group (GGAL) was traded in all the days of trading sessions. So, we have decided to do not construct a portfolio with only one security.

Table 2: Number of assets in the portfolios formed by the Markowitz model

Year of Portfolio Selection	BCBA	BCS	BMV	BOVESPA
1995	4	7	3	4
1996	3	8	9	12
1997	3	5	7	7
1998	4	3	7	6
1999	4	5	8	7
2000	4	9	10	13
2001		5	6	8
2002	5	4	11	11
2003	6	6	15	20
2004	4	14	13	12
2005	6	12	17	20
2006	8	16	10	16
2007	9	12	14	20
2008	13	9	9	40
2009	8	14	11	37
2010	10	25	14	44
2011	8	9	12	31

Source: Prepared by the authors.

By analyzing Table 2, it can be seen that the overall securities number of each

portfolio increases throughout the time. Calculating the correlations between the years and amount of the securities, we have that the correlation in the Argentina Stock Exchange is 0.83, in the Chile Stock Exchange is 0.65, in the Mexico Stock Exchange is 0.65 and the Brazil Stock Exchange is 0.86. The year 2001 was disregarded for the Argentina for the reason of the absence of portfolio. Table 3 shows the quantity of securities per country used in the Markowitz model for which was used to verify the correlations.

The correlations between the securities number of each country versus the securities quantity in each portfolio were the followings: the correlation in the Argentina Stock Exchange was 0.56, in the Chile Stock Exchange was 0.78 in the Mexico Stock Exchange was 0.57 and in the Brazil Stock Exchange was 0.93. The overall correlation between the securities number in a given Stock Exchange versus the amount of securities in the portfolio was 0.91.

This proves the prerequisite of a prior selected method that Markowitz (1952) called of first stage in the portfolio selection process, which is a deliberate investor decision, because besides the results are divergent from the results of the researches conducted by Evans and Archer (1968), Wagner and Lau (1971), Brito (1989) and Ceretta and Costa Jr. (2000), for instance, a large amount of securities in the portfolio increases the transaction costs.

Table 3: Number of active per bag used in the Markowitz model

Year of Portfolio Selection	BCBA	BCS	BMV	BOVESPA
1995	12	13	3	12
1996	13	13	15	12
1997	6	8	15	17
1998	7	5	15	10
1999	11	5	19	16
2000	5	9	20	29
2001	1	5	13	31
2002	6	6	18	35
2003	11	10	21	38
2004	10	20	24	43
2005	8	25	24	50
2006	12	27	32	60
2007	14	36	38	93
2008	14	27	37	125
2009	13	32	44	138
2010	19	44	41	158
2011	27	43	48	162

Source: Prepared by the authors.

On the other hand, the model developed in this research covers the two stages defined by Markowitz (1952). Thus, regardless of the number of securities on the stock market, the model will select the necessary quantity to stabilize the risk eliminating the idiosyncratic risk. In addition, the amount of securities selected in the main four stock exchanges in Latin America was within the limit of 15 assets, which

was the number found by prior studies.

Conclusion

The Modern Portfolio Theory had its beginning in the 1950s. One of the most important researches which have contributed to the development of the Finance Theory was the Harry Markowitz paper in 1952, which has developed a portfolio selection model by minimizing the risk of the portfolio. The great merit of Markowitz was the development of a mathematical model to calculate the portfolio risk, considering the assets covariance. Thus, the Markowitz model has achieved which none other model had reached: to combine assets so that one other asset is put or removed of the portfolio, its risk influences all other risks and the overall portfolio risk.

On the conclusion of his paper, Markowitz (1952, p. 91) assume “that better methods, which take into account more information, can be found”. For Markowitz, which “is needed is essentially a ‘probabilistic’ reformulation of security analysis” which he considered as part of the first stage in the portfolio selection process.

In an attempt to develop a model for first-stage portfolio selection process, we have begun a study of financial indicators that were able to predict the value generation capacity of the companies, which are called of value drivers. The belief underlying of our research was that the securities which had in their idiosyncrasy the ability of generating value would be the best options to be used in the Markowitz model. Throughout this research, it was realized that researchers have been studying it for over 50 years and they are still studying.

Furthermore, it was noticed that instead of proposing a filter for assets selecting that would be used in the Markowitz model, we could develop a model to select the assets and construct the portfolio, by determining their weights and the amount of assets that would be part of the portfolio, creating thus an alternative model to the Markowitz model.

The underlying issue in this objective was that a stock portfolio selected through value drivers would get the benefits of diversification with a lower amount of securities when compared to the amount selected by the Markowitz (1952) model. To achieve this goal, we first identified the key value drivers, studied for over 40 years. Then, we have developed a model to select the assets and to construct the portfolios by calculating the amount of securities of each portfolio.

Given the results, it appears that the hypothesis that a stock portfolio selected through value drivers reach the benefits of diversification with a lower amount of assets, when compared to the quantity selected by the Markowitz model cannot be rejected.

During this study, some limitations, difficulties, findings and ideas have emerged. From these, some proposals for future studies have been arisen. These propositions can be divided into three categories: theoretical suggestions; empirical suggestions; and applied suggestions.

The theoretical suggestions are directly linked to the proposed model and require a deeper knowledge of the researcher. One suggestion is to use generalized linear models rather than a β_0 -equal-to-zero multiple regression, as was done in this study. The advantage of generalized linear models is that the researcher has the free-

dom to determine (or to test) the random errors distribution, which was not possible with the use of linear regression. Thus, the regression betas could be better estimated, which could result in a better result.

Another theoretical suggestion is to use a cause-effect model to determine the value drivers which will be used. Using a logit model appears to be a good option, considering that this model is stochastic and one of the properties of stochastic models is Independence of Irrelevant Alternatives (IIA). In addition, this model is multivariate and aims to identify the relative importance of a set of independent variables on a dependent variable.

Regarding the empirical suggestion, the use of other value drivers is recommended, both in quality and quantity. As regards to the applied suggestion, it is suggested to replicate this study in other capital markets, particularly in the mature markets and in developed countries. Still about the applied suggestion, it is possible to test the period that the proposed model performs better than the Markowitz model over time.

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